TRINITY COLLEGE

**Semester One Examination, 2019**

**Question/Answer booklet**

# MATHEMATICS SPECIALIST UNIT 1

## Section One: Calculator-free

# SOLUTIONS

Student number: In figures | | | | | | | | |

In words _____

Your name _____

## Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**See next page**

**Section One: Calculator-free** **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

---

**Question 1** **(8 marks)**

Let $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j}$, $\mathbf{b} = -4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c} = -\mathbf{i} + 2\mathbf{j}$.

(a) Determine

(i) $\mathbf{b} - \mathbf{c}$. (1 mark)

| Solution |
|---|
| $\begin{pmatrix}-4\\3\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}-3\\1\end{pmatrix}$ |
| **Specific behaviours** |
| ✓ correct vector |

(ii) $5\mathbf{c} + 3\mathbf{a}$. (2 marks)

| Solution |
|---|
| $5\begin{pmatrix}-1\\2\end{pmatrix} + 3\begin{pmatrix}3\\-5\end{pmatrix} = \begin{pmatrix}-5\\10\end{pmatrix} + \begin{pmatrix}9\\-15\end{pmatrix} = \begin{pmatrix}4\\-5\end{pmatrix}$ |
| **Specific behaviours** |
| ✓ determines scalar multiples<br>✓ correct vector |

(iii) $|\mathbf{a} - \mathbf{c}|$. (2 marks)

| Solution |
|---|
| $\begin{pmatrix}3\\-5\end{pmatrix} - \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}4\\-7\end{pmatrix}$<br>$\sqrt{4^2 + (-7)^2} = \sqrt{65}$ |
| **Specific behaviours** |
| ✓ determines sum<br>✓ correct value |

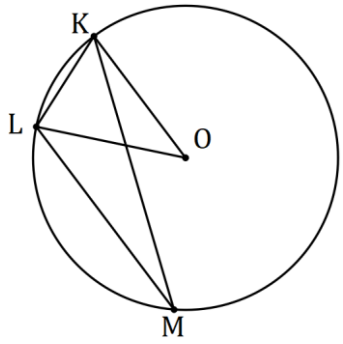
(b) Determine a unit vector that is parallel to $\mathbf{a} + \mathbf{c}$ but in the opposite direction. (3 marks)

| Solution |
|---|
| $-\left(\begin{pmatrix}3\\-5\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix}\right) = \begin{pmatrix}-2\\3\end{pmatrix}$<br>$\left\lvert\begin{pmatrix}-2\\3\end{pmatrix}\right\rvert = \sqrt{13}$<br>$\text{Soln: } \frac{1}{\sqrt{13}}\begin{pmatrix}-2\\3\end{pmatrix}$ |
| **Specific behaviours** |
| ✓ determines $-(\mathbf{a} + \mathbf{c})$<br>✓ determines magnitude<br>✓ correct unit vector |

**See next page**

**Question 2** **(4 marks)**

In the diagram below (not drawn to scale) $K, L$ and $M$ lie on the circle with centre $O$ and $OK$ is parallel to $ML$.



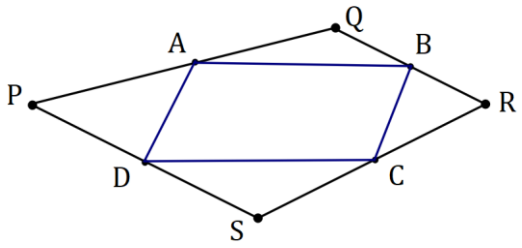

Determine, with reasons, the size of $\angle OLK$ and the size of $\angle KLM$ when $\angle OKM = 14°$.

| **Solution** |
|---|
| $\angle KML = \angle OKM = 14$ (Alternate angles)<br><br>$\angle KOL = 2 \times \angle KML = 28$ (Angle at centre)<br><br>$\angle OLK = (180 - 28) \div 2 = 76°$ (Isosceles)<br><br>$\angle OLM = \angle KOL = 28$ (Alternate angles)<br><br>$\angle PQR = 76 + 28 = 104°$ |
| **Specific behaviours** |
| ✓ $\angle ACB$ with reason<br>✓ $\angle AOB$ with reason<br>✓ $\angle OQP$ with reason<br>✓ $\angle PQR$ |

**See next page**

**Question 3** **(7 marks)**

Quadrilateral $PQRS$ is shown below. The midpoints of sides $PQ, QR, RS$ and $SP$ are $A, B, C$ and $D$ respectively. Let $\overrightarrow{PQ} = 2\mathbf{q}, \overrightarrow{PR} = 2\mathbf{r}$ and $\overrightarrow{PS} = 2\mathbf{s}$.




| Solution (a) |
|---|
| See diagram |
| **Specific behaviours** |
| ✓ correct quadrilateral |

(a) Sketch quadrilateral $ABCD$ on the diagram above. (1 mark)

(b) Determine expressions for $\overrightarrow{PB}, \overrightarrow{PC}$ and $\overrightarrow{BC}$ in terms of $\mathbf{q}, \mathbf{r}$ and $\mathbf{s}$. (3 marks)

| Solution |
|---|
| $\overrightarrow{PB} = 2\mathbf{q} + \frac{1}{2}(2\mathbf{r} - 2\mathbf{q}) = \mathbf{q} + \mathbf{r}$<br><br>$\overrightarrow{PC} = 2\mathbf{s} + \frac{1}{2}(2\mathbf{r} - 2\mathbf{s}) = \mathbf{s} + \mathbf{r}$<br><br>$\overrightarrow{BC} = \overrightarrow{BP} + \overrightarrow{PC}$<br>$= (-\mathbf{q} - \mathbf{r}) + (\mathbf{s} + \mathbf{r}) = \mathbf{s} - \mathbf{q}$ |
| **Specific behaviours** |
| ✓ derives expression for $\overrightarrow{PB}$<br>✓ derives expression for $\overrightarrow{PC}$<br>✓ derives expression for $\overrightarrow{BC}$ |

(c) Prove that $\overrightarrow{AD} = \overrightarrow{BC}$ and $\overrightarrow{AB} = \overrightarrow{DC}$. (3 marks)

| Solution |
|---|
| $\overrightarrow{AD} = \overrightarrow{PD} - \overrightarrow{PA}$<br>$= \mathbf{s} - \mathbf{q}$<br>$= \overrightarrow{BC}$<br><br>$\overrightarrow{AB} = \overrightarrow{PB} - \overrightarrow{PA}$<br>$= \mathbf{q} + \mathbf{r} - \mathbf{q}$<br>$= \mathbf{r}$<br><br>$\overrightarrow{DC} = \overrightarrow{PC} - \overrightarrow{PD}$<br>$= \mathbf{s} + \mathbf{r} - \mathbf{s}$<br>$= \mathbf{r}$<br>$= \overrightarrow{AB}$ |
| **Specific behaviours** |
| ✓ derives expression for $\overrightarrow{AD}$ and equates to $\overrightarrow{BC}$<br>✓ derives expression for $\overrightarrow{AB}$<br>✓ derives expression for $\overrightarrow{DC}$ and equates to $\overrightarrow{AB}$ |

**Question 4** **(6 marks)**

(a) Body $A$ moves with a velocity of $6\sqrt{2}\mathbf{i} - 6\sqrt{2}\mathbf{j}$ ms$^{-1}$. Determine the speed of this body and the bearing it is travelling in. (3 marks)

| **Solution** |
|---|
| $s^2 = \left(6\sqrt{2}\right)^2 + \left(-6\sqrt{2}\right)^2$<br>$s = 12 \text{ m/s}$<br><br>$\angle(x\text{-axis}) = -45°$<br><br>$\text{Bearing} = 90 + 45 = 135°$ |
| **Specific behaviours** |
| ✓ correct speed<br>✓ angle with $x$-axis<br>✓ correct bearing |

(b) Body $B$ moves $32$ m on a bearing of $300°$. Express this displacement in component form using unit vectors **i** and **j**. (3 marks)

| **Solution** |
|---|
| $\angle(x\text{-axis}) = 150°$<br><br>$\mathbf{r} = 32\cos(150°)\,\mathbf{i} + 32\sin(150°)\,\mathbf{j}$<br>$= -16\sqrt{3}\mathbf{i} + 16\mathbf{j}$ |
| **Specific behaviours** |
| ✓ correct angle from $x$-axis<br>✓ correct **i**-coefficient<br>✓ correct **j**-coefficient |

**See next page**

**Question 5** **(7 marks)**

(a) The work done, in joules, by a force of $\mathbf{F}$ Newtons in changing the displacement of an object by $\mathbf{s}$ metres, is given by the scalar product of $\mathbf{F}$ and $\mathbf{s}$. Determine the work done by

(i) force $\mathbf{F} = (5\mathbf{i} + 10\mathbf{j})$ N that moves a small body from $(16\mathbf{i} - 2\mathbf{j})$ m to $(22\mathbf{i} + 8\mathbf{j})$ m. (2 marks)

| **Solution** |
|---|
| $\begin{pmatrix}22\\8\end{pmatrix} - \begin{pmatrix}16\\-2\end{pmatrix} = \begin{pmatrix}6\\10\end{pmatrix}$<br><br>$w = \begin{pmatrix}5\\10\end{pmatrix} \cdot \begin{pmatrix}6\\10\end{pmatrix} = 30 + 100 = 130 \text{ J}$ |
| **Specific behaviours** |
| ✓ displacement vector<br>✓ correct work done |

(ii) a horizontal force of $45$ N that pushes a small body $0.4$ m up a slope inclined at $45°$ to the horizontal. (2 marks)

| **Solution** |
|---|
| $w = 45 \times 0.4 \times \cos 45$<br>$= 45 \times 0.4 \times \frac{\sqrt{2}}{2}$<br>$= 9\sqrt{2} \text{ J}$ |
| **Specific behaviours** |
| ✓ uses correct expression<br>✓ correct work done |

(b) Determine the vector projection of $(-\mathbf{i} - 4.5\mathbf{j})$ on $(3\mathbf{i} - 4\mathbf{j})$. (3 marks)

| **Solution** |
|---|
| $\begin{pmatrix}-1\\-4.5\end{pmatrix} \cdot \begin{pmatrix}3\\-4\end{pmatrix} = 15$<br><br>$\begin{pmatrix}3\\-4\end{pmatrix} \cdot \begin{pmatrix}3\\-4\end{pmatrix} = 25$<br><br>$\frac{15}{25}\begin{pmatrix}3\\-4\end{pmatrix} = \begin{pmatrix}9/5\\-12/5\end{pmatrix}$ |
| **Specific behaviours** |
| ✓ scalar products<br>✓ substitutes into expression<br>✓ correct vector projection |

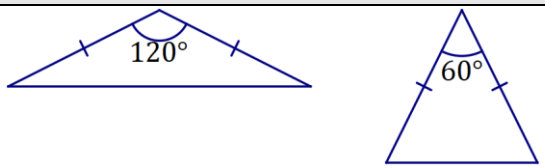
**See next page**

**Question 6** **(6 marks)**

Consider the following statement that refers to two **isosceles** triangles.

If the triangles have the same area, then the triangles are congruent.

(a) Use a counter-example to explain why the statement is false. (2 marks)

| **Solution** |
|---|
|  <br>The isosceles triangles shown have the same area but are not congruent. |
| **Specific behaviours** |
| ✓ sketches two isosceles triangles<br>✓ shows supplementary angles and congruent sides<br>Or ✓ shows dimensions that give same area |

(b) Write the contrapositive statement and state whether it is true or false. (2 marks)

| **Solution** |
|---|
| If the triangles are not congruent, then the triangles do not have the same area.<br><br>This statement is false. |
| **Specific behaviours** |
| ✓ correct contrapositive statement<br>✓ states false |

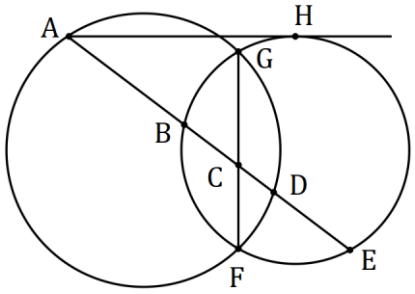
(c) Write the inverse statement and state whether it is true or false. (2 marks)

| **Solution** |
|---|
| If the triangles do not have the same area, then the triangles are not congruent.<br><br>This statement is true. |
| **Specific behaviours** |
| ✓ correct inverse statement<br>✓ states true |

**See next page**

**Question 7** **(6 marks)**

In the diagram below (not drawn to scale), two circles intersect at $F$ and $G$. $AH$ is a tangent to the circle at $H$. $AE$ is a straight line that cuts the circles at $A, B, D$ and $E$ and intersects chord $GF$ at $C$. $AB = 4, GC = 5.5, CF = 2$, $AH = 8$ and $BC < CE$.




(a) Deduce that $BE = 12$. (2 marks)

| Solution |
|---|
| $AH^2 = AB \times AE$<br>$AE = 8^2 \div 4 = 16$<br><br>$BE = AE - AB$<br>$= 16 - 4 = 12$ |
| **Specific behaviours** |
| ✓ justifies length of $AE$<br>✓ justifies length of $BE$ |

(b) Determine $BC$ and $CD$, justifying your answers. (4 marks)

| Solution |
|---|
| $BC \times CE = GC \times CF$<br>$x = BC$<br>$x(12 - x) = 5.5 \times 2 = 11$<br>$x^2 - 12x + 11 = 0$<br>$(x - 1)(x - 11) = 0$<br>$x = BC = 1$<br><br>$AC \times CD = GC \times CF = 11$<br>$CD = 11 \div (4 + 1) = {}^{11}/_{5}$ |
| **Specific behaviours** |
| ✓ justifies equation for $BC$<br>✓ length of $BC$<br>✓ justifies equation for $CD$<br>✓ length of $CD$ |

**See next page**

**Question 8** **(8 marks)**

(a) Evaluate $\frac{^{55}P_2}{^{6}P_3}$. (3 marks)

| **Solution** |
|---|
| $\frac{^{55}P_2}{^{6}P_3} = \frac{55!}{53!} \div \frac{6!}{3!}$<br>$= \frac{55 \times 54}{6 \times 5 \times 4}$<br>$= \frac{99}{4}$ |
| **Specific behaviours** |
| ✓ expresses as factorials<br>✓ eliminates factorials<br>✓ correct value, simplified |

(b) Given that $^{n}P_{r+1} = k \times {}^{n}P_r$, determine the constant $k$ in terms of $n$ and/or $r$. (3 marks)

| **Solution** |
|---|
| $^{n}P_{r+1} = \frac{n!}{(n-r-1)!}$<br>$= \frac{(n-r)n!}{(n-r)(n-r-1)!}$<br>$= \frac{(n-r)n!}{(n-r)!}$<br>$= (n-r) \times {}^{n}P_r$<br><br>$\therefore k = n - r$ |
| **Specific behaviours** |
| ✓ expresses LHS using factorials<br>✓ factors out term from denominator<br>✓ correct expression |

(c) Given that $^{13}P_8 = 51\,891\,840$, determine $^{13}P_{10}$. (2 marks)

| **Solution** |
|---|
| $^{13}P_{10} = 4 \times {}^{13}P_9 = 4 \times 5 \times {}^{13}P_8 = 20 \times {}^{13}P_8$<br><br>$20 \times 51891840 = 1\,037\,836\,800$ |
| **Specific behaviours** |
| ✓ correct multiplier<br>✓ correct value |

**End of questions**

Supplementary page

Question number: _____